UFMG

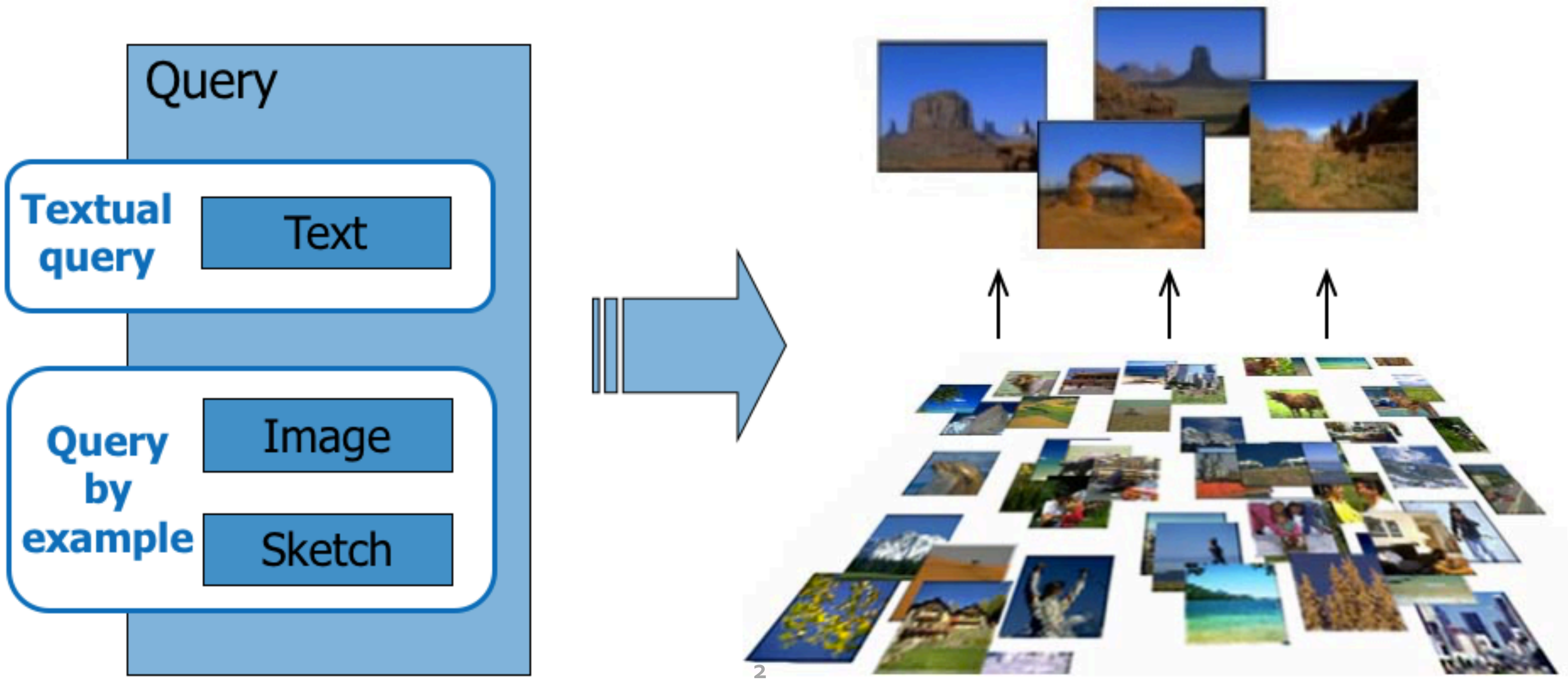
PATREO — Pattern Recognition for Earth Observation — DCC - UFMG

Locality Sensitive Hashing (LSH)

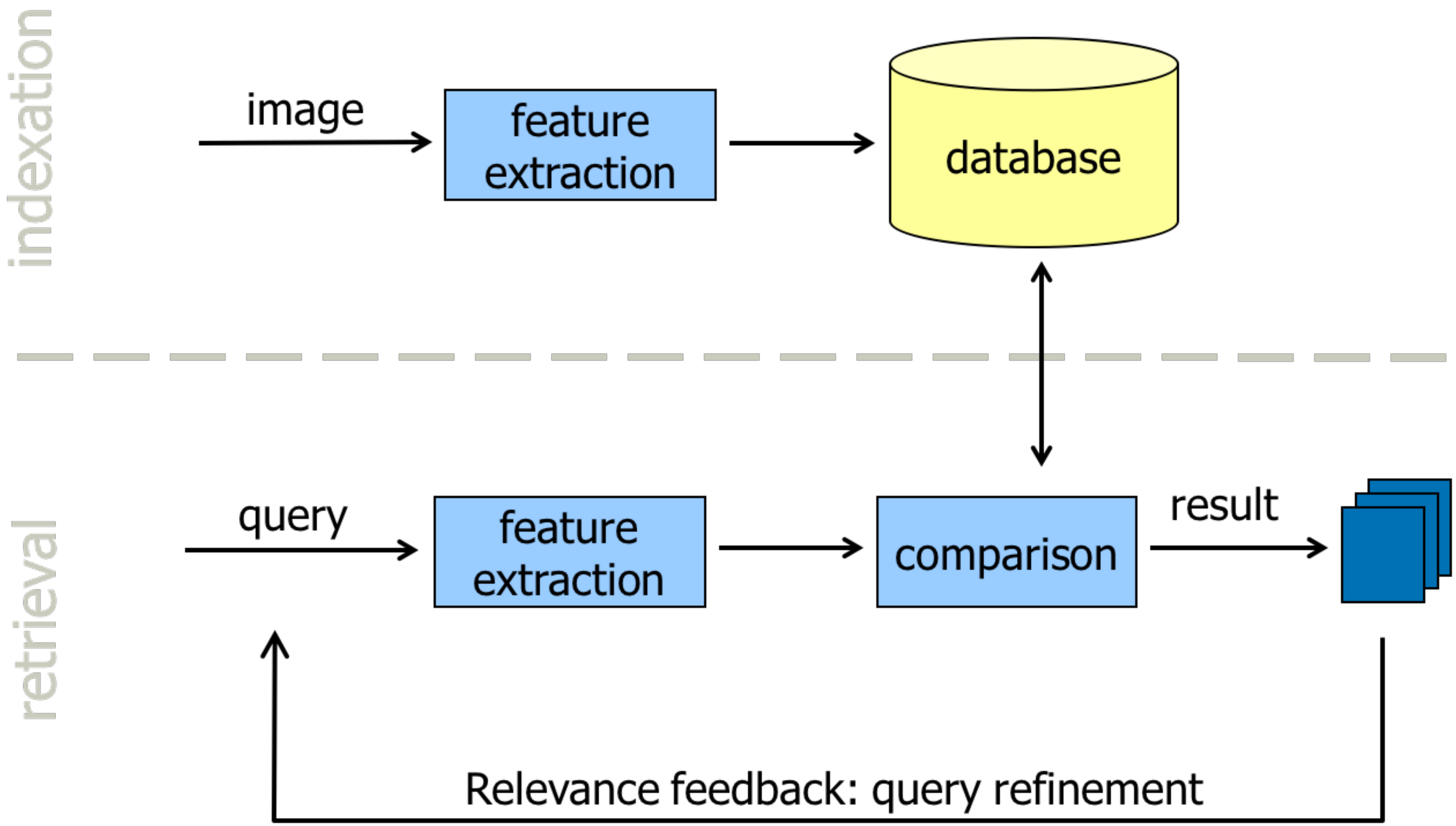
Eduardo Tavares

Motivation

- ✓ Description Based Image Retrieval (DBIR)
- ✓ Content Based Image Retrieval (CBIR)

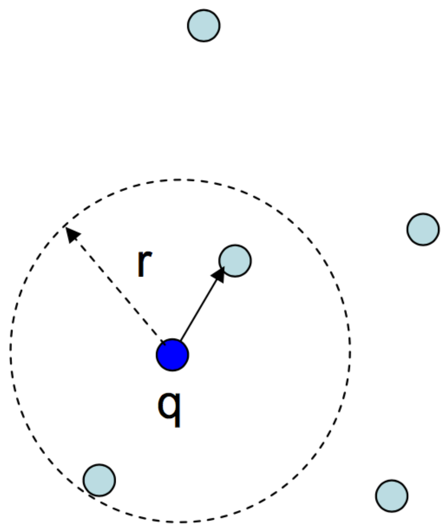


Common components of CBIR system



Nearest Neighbour Problem

- ✓ Given: a set P of point in R^d
- ✓ **Nearest Neighbour**: for any query q, returns a point $p \in P$ minimizing D(p,q)
- ✓ **r-Near Neighbour**: for any query q, returns a point $p \in P$ such that $D(p,q) \leq r$ (if it exists)



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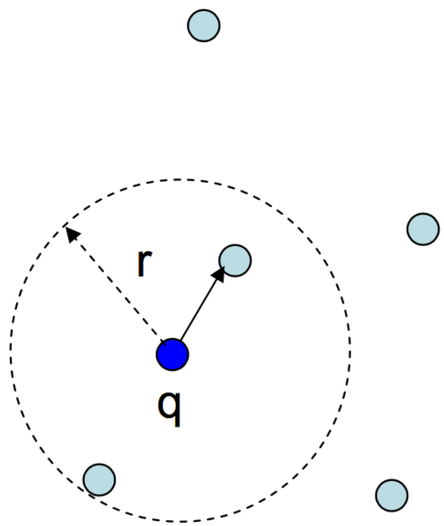
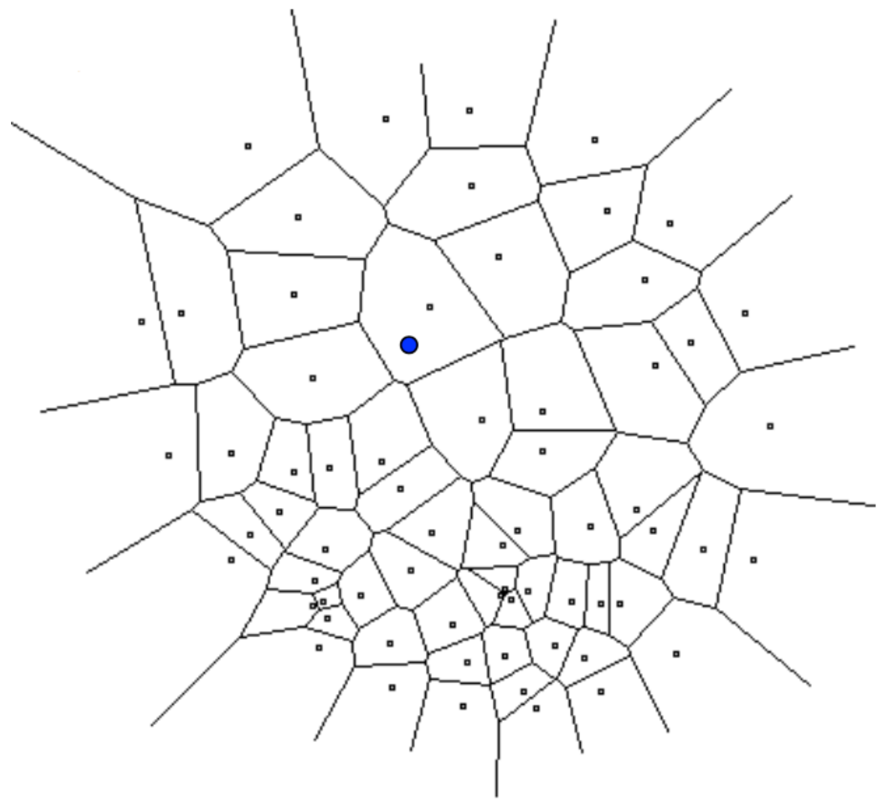


DEFINITION (ℓ_p DISTANCE). *The distance between any two d-dimensional points $\vec{o}$ and $\vec{q}$ in the ℓ_p space, denoted as $\ell_p(\vec{o}, \vec{q})$, is computed as:*

$$\ell_p(\vec{o}, \vec{q}) = \sqrt[p]{\sum_{i=1}^{d} |o_i - q_i|^p}$$

The case of d = 2

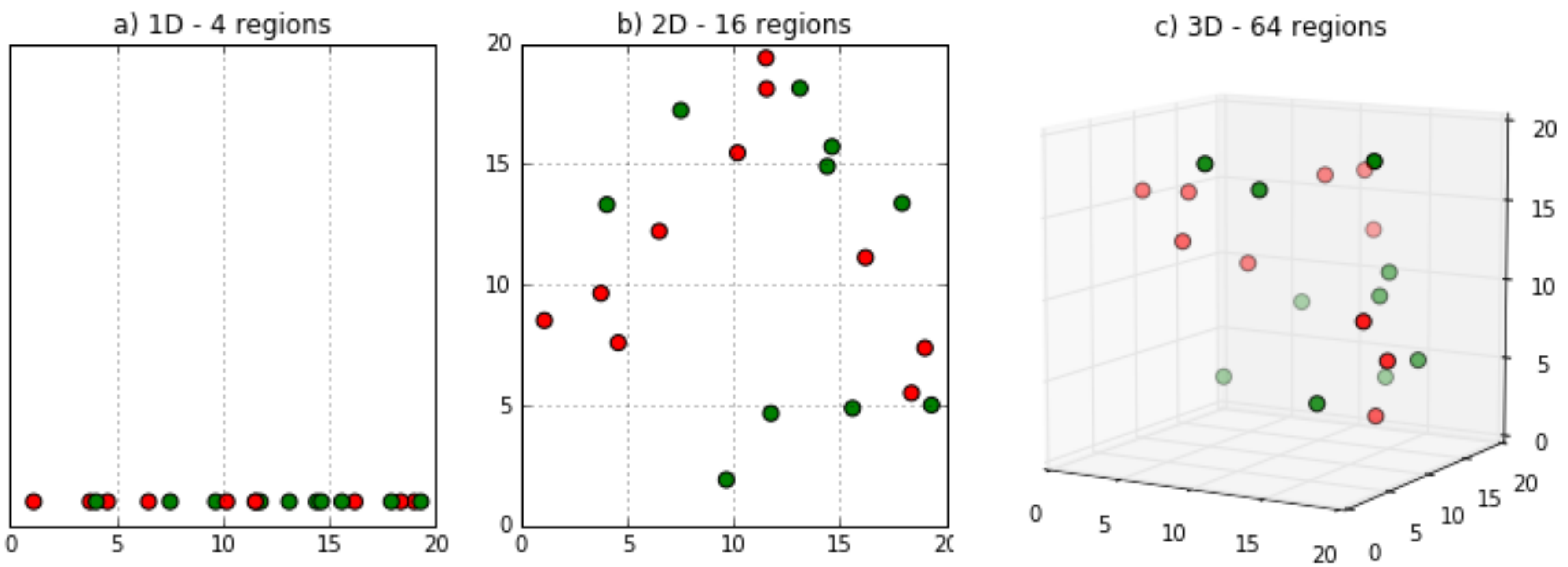
- ✓ Compute **Voronoi diagram**
- ✓ Given q, perform point location
- ✓ Performance:
  - ✓ Space: O(n)
  - ✓ Query time: O(log n)



The case of d > 2

- Voronoi diagram has size $O(n^d)$
- Another possibility: linear scan ($O(dn)$ time)
- That's all that is known about exact algorithms with theoretical guarantees.
- In practice:
  - Kd-trees work "well" in "low-medium" dimensions: require sublinear time and near linear space for d < 10 - 20
  - Time or space requirements grow exponentially in the dimension.

Problems: curse of dimensionality



The cost of exact matching

- Finding the 10-NN of 1000 distinct queries in 1 million vectors
  - Assuming 128-D Euclidian descriptors
  - i.e., 1 billion distances, computed on a 8-core machine

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- Finding the 10-NN of 1000 distinct queries in 1 million vectors
  - Assuming 128-D Euclidian descriptors
  - i.e., 1 billion distances, computed on a 8-core machine
- 5.5 seconds

- Hamming distance: 1000 queries, 1M database vectors:
  - Computing the 1 billion distances: 0.6 second

Hamming Space

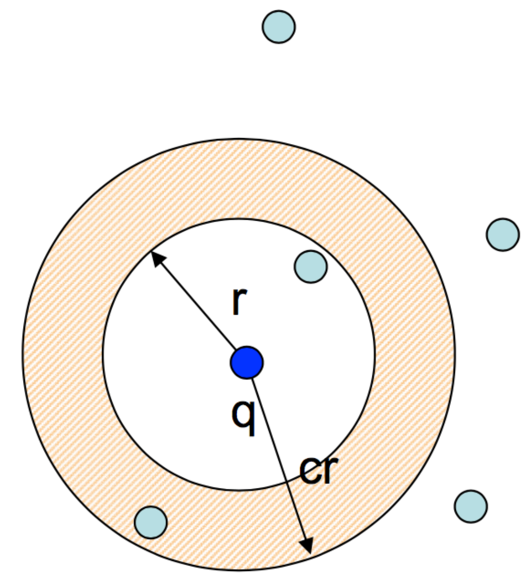
- **Definition:** *Hamming space* is the set of all 2^N binary strings of length N.
- **Definition:** The *Hamming distance* between two equal length binary strings is the number of positions for which the bits are different.
- $||1011101; 1001001||_H = 2$
- $||1110101; 1111101||_H = 1$

The cost of exact matching

- To improve the scalability: find the nearest neighbour in probability only: **Approximate nearest neighbour (ANN) search**
- An approximate nearest neighbour should suffice in most cases.

Approximate Near Neighbour

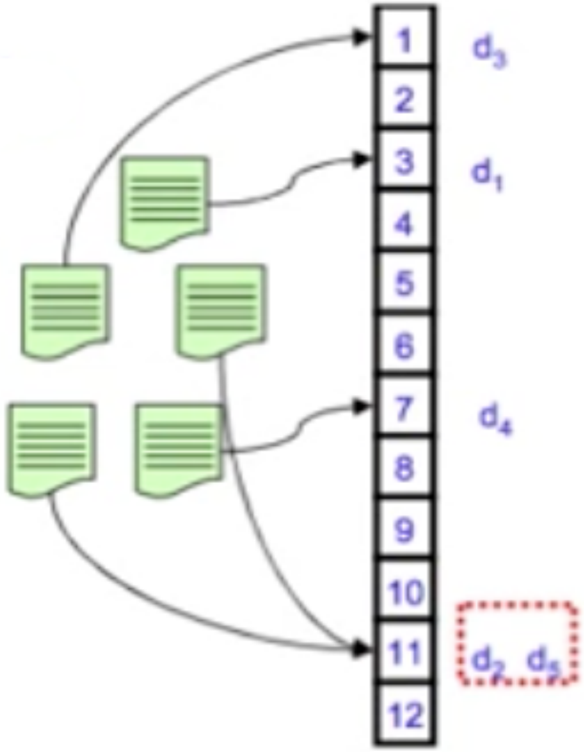
- c-Approximate r-Near Neighbour: build data structure which, for any query q:
  - If there is a point $p \in P$, $||p - q|| \leqq r$
  - it returns $p' \in P$, $||p' - q|| \leqq cr$



- Three (contradictory) performance criteria for ANN schemes:
  - Search quality (retrieved vectors are actual nearest neighbours)
  - Speed
  - Memory usage

Hashing

- Hash function: any function $\mathcal{H}$ which has, as a minimum, the following two properties:
  - Compression
  - ease of computation



Locality Sensitive Hashing

- Idea: project the data into a low-dimensional binary (Hamming) space; while preserving some properties from the original space.

- Construct hash functions $h: R^d \rightarrow U$ such that for any points p, q:
  - If $D(p,q) \leq r$, then $Pr[h(p) = h(q)]$ is "high".
  - If $D(p,q) > cr$, then $Pr[h(p) = h(q)]$ is "small".

- Then we can solve the problem by performing hash table lookup.
- LSH is a general framework; for a given D we need to find the right h.

LSH [Indyk-Motwani'98]

- A family $\mathcal{H}$ of functions $h: R^d \rightarrow U$ is called (P_1, P_2, r, cr)-sensitive , if for any p, q:
  - If $D(p,q) < r$, then $Pr[h(p) = h(q)] > P_1$
  - If $D(p,q) > cr$, then $Pr[h(p) = h(q)] < P_2$.
- $P_1 > P_2$.

Bit sampling

- ✓ Works for the Hamming distance over d-dimensional vectors $\{0,1\}^d$.
- ✓ The family $\mathcal{H}$ of hash functions is simply the family of all the projections of points in one of the d coordinates.
- ✓ $\mathcal{H} = \{h : \{0,1\}^d \rightarrow \{0,1\} \mid h(x) = x_i$ for some $i \in \{1, \ldots, d\}$, where x_i is the *i*th coordinate of x.
- ✓ A random function h from $\mathcal{H}$ simply selects a random bit from the input point.

Algorithm: preprocessing

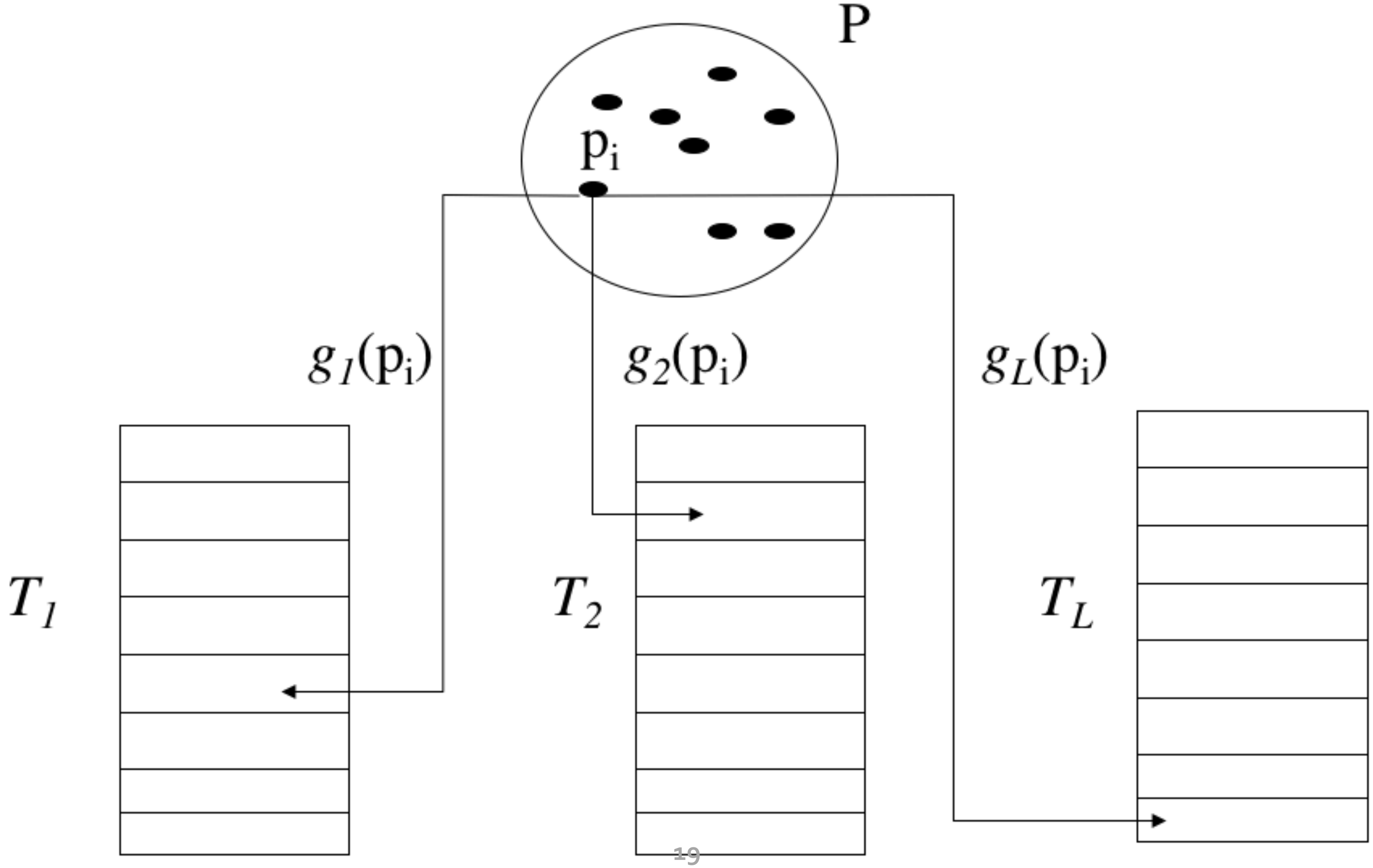
- Hash the data-point using several LSH functions ($g_i(.) = \{ h_1(.) \ldots h_k(.) \}$) so that probability of collision is higher for closer objects

- Input
  - Set of N points $\{ p_1, \ldots\ldots\ldots p_n \}$
  - L (number of hash tables)
- Output
  - Hash tables T_i , $i = 1, 2, \ldots. L$
- Foreach $i = 1, 2, \ldots. L$
  - Initialize T_i with a random hash function $g_i(.)$
- Foreach $i = 1, 2, \ldots. L$

  Foreach $j = 1, 2, \ldots. N$

  Store point p_j on bucket $g_i(p_j)$ of hash table T_i

Algorithm: preprocessing



Algorithm : cr - NNS Query

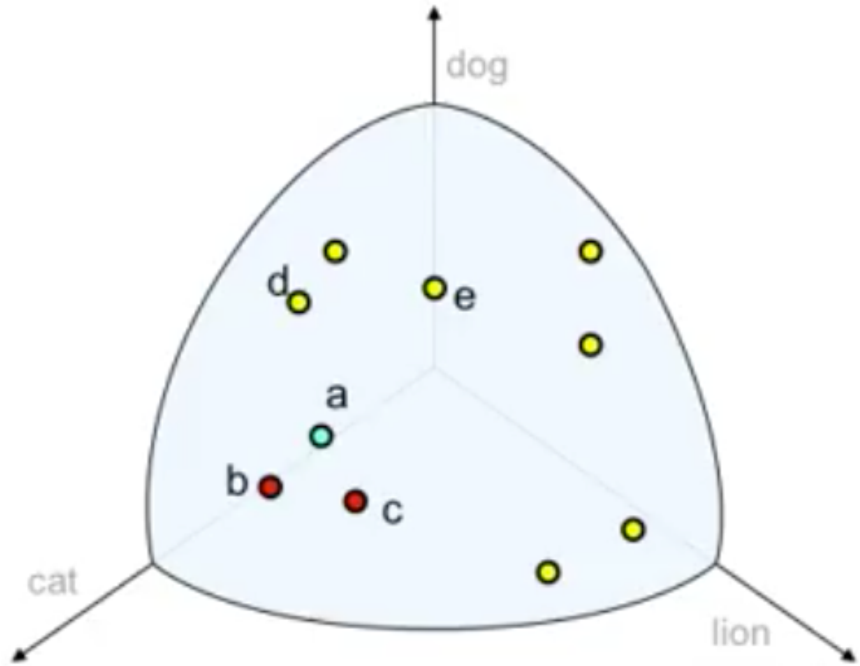
- Input
  - Query point q
  - K (number of approx. nearest neighbors)
- Access
  - Hash tables T_i , $i = 1$, 2, …. L
- Output
  - Set S of K (or less) approx. nearest neighbors
- S $\leftarrow \varnothing$

  Foreach $i = 1$, 2, …. L
  - S $\leftarrow$ S $\cup$ { points found in $g_i(q)$ bucket of hash table T_i }

Random hyperplane based LSH

- ✓ Selects b hash functions $h_r(\cdot)$, each of which simply rounds the output of the product of x with a random hyperplane defined by a random vector r:

$$h_r(x) = \begin{cases} 1 & if\ r^T x \geq 0 \\ 0 & otherwise \end{cases}$$



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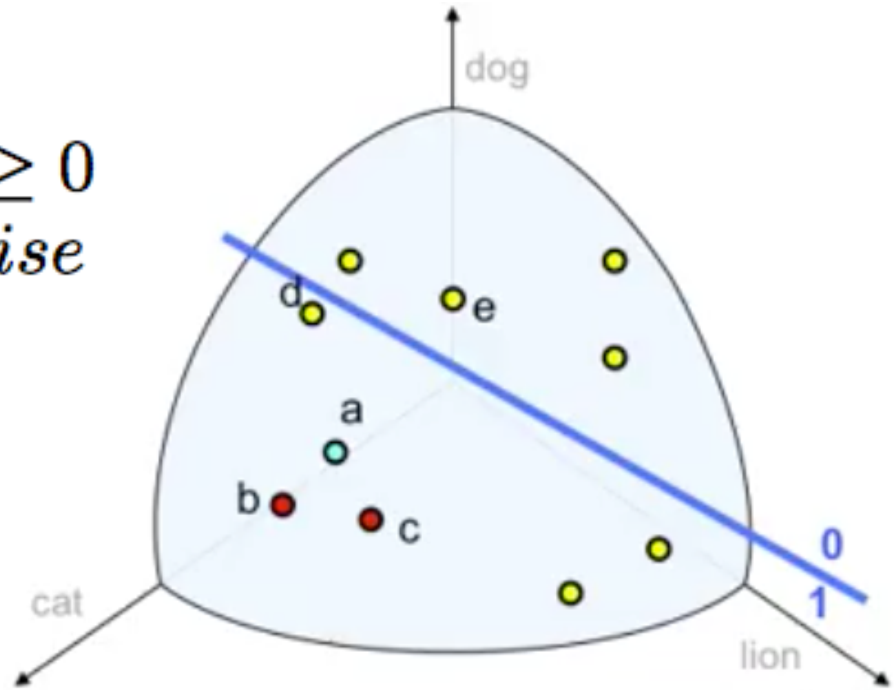

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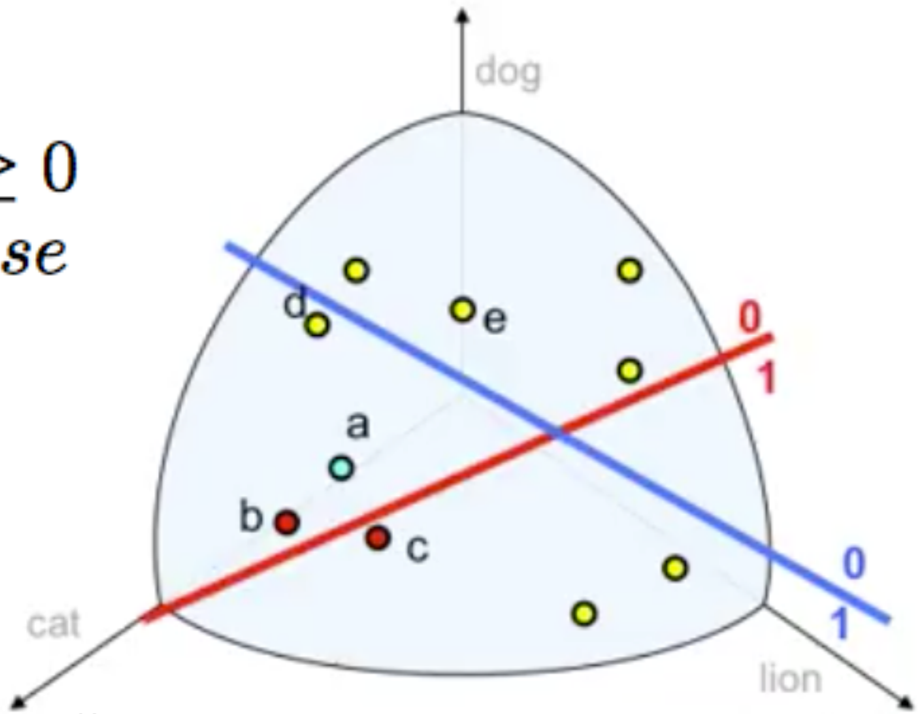

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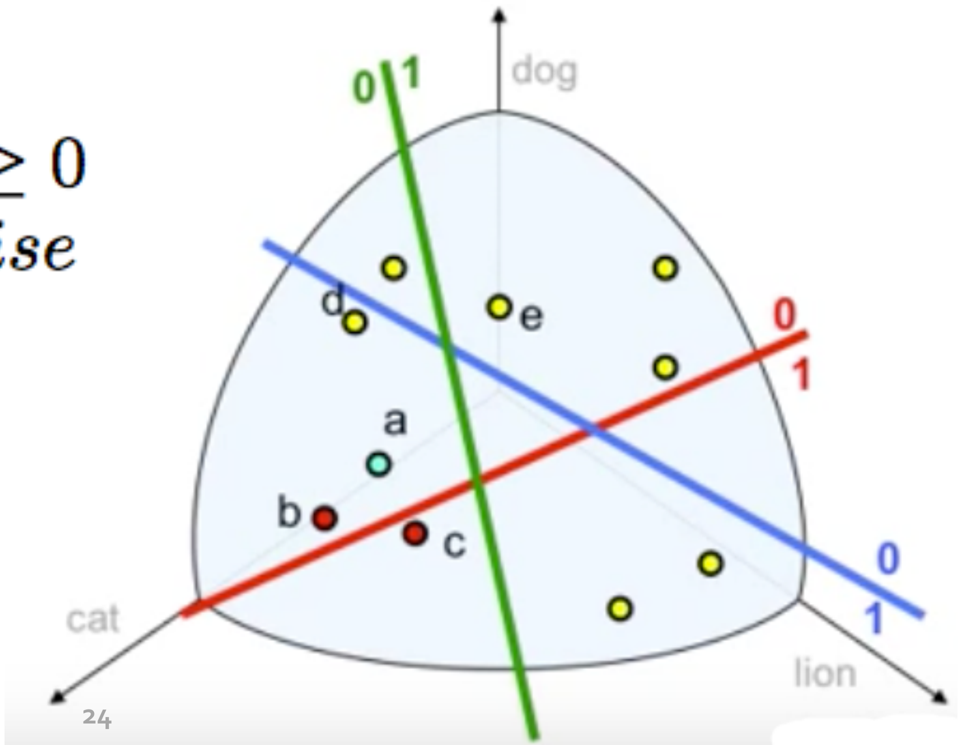
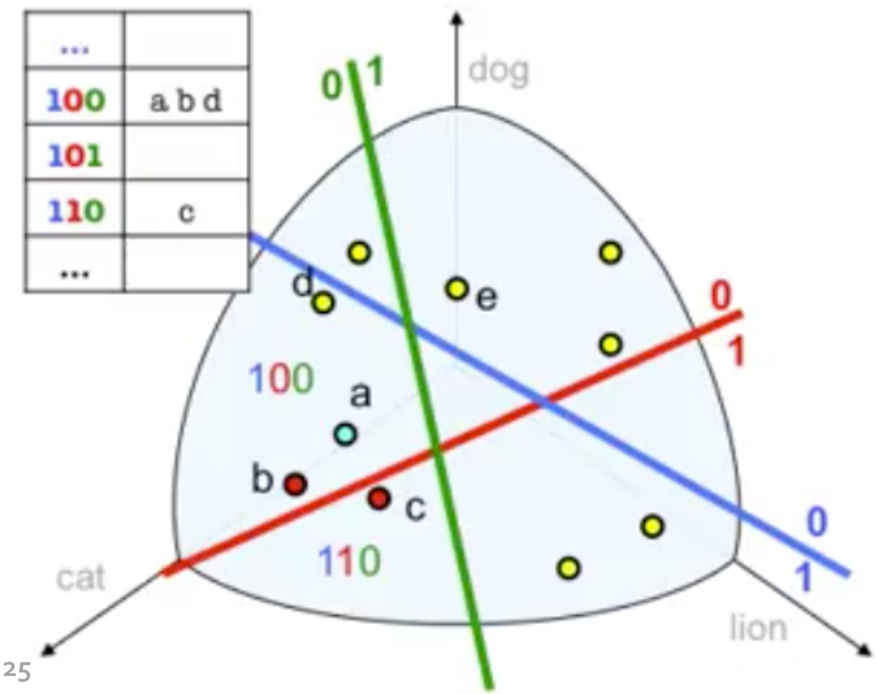


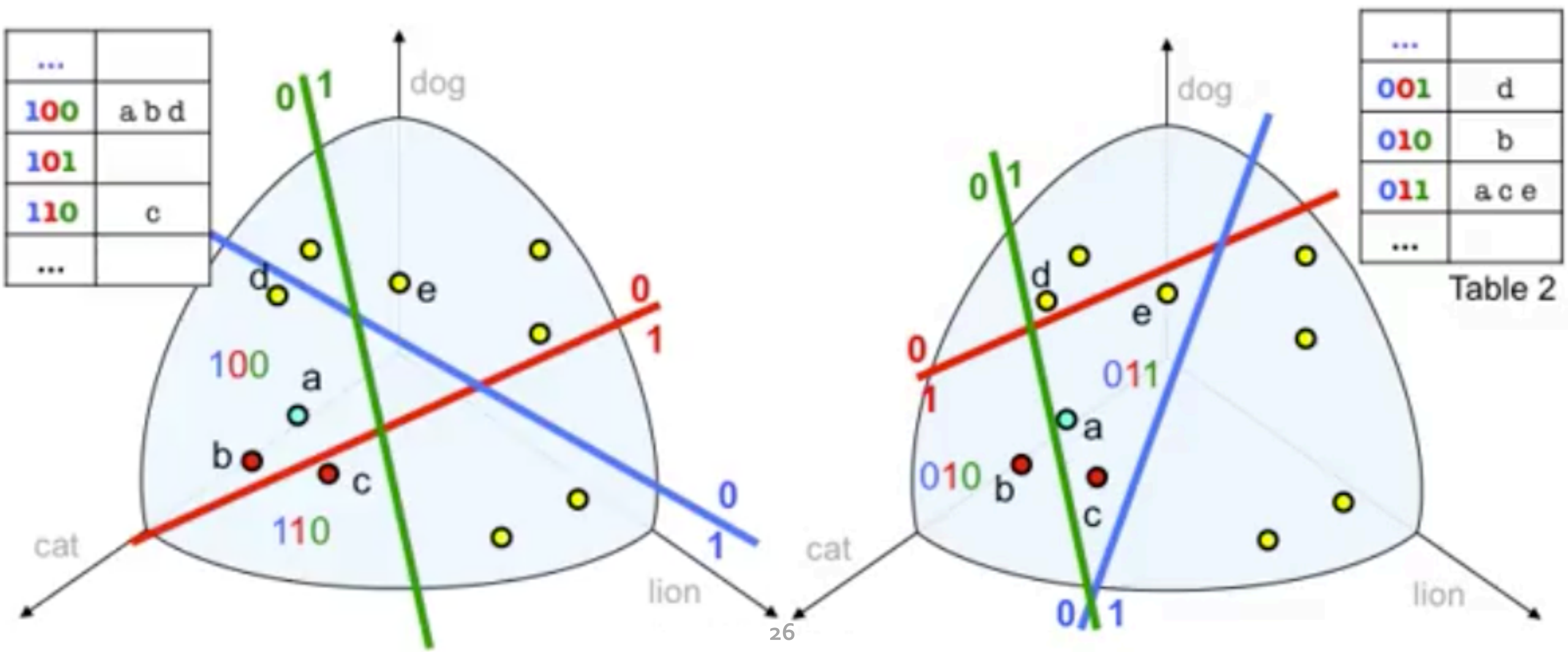
Random hyperplane based LSH

- Compare **a** to points with same hash-code
  - **b** … indeed similar to a
  - **d** … false positive, will be eliminated
  - **c** … different hash-code, will miss it



Random hyperplane based LSH

- ✓ Repeat with different hyperplanes

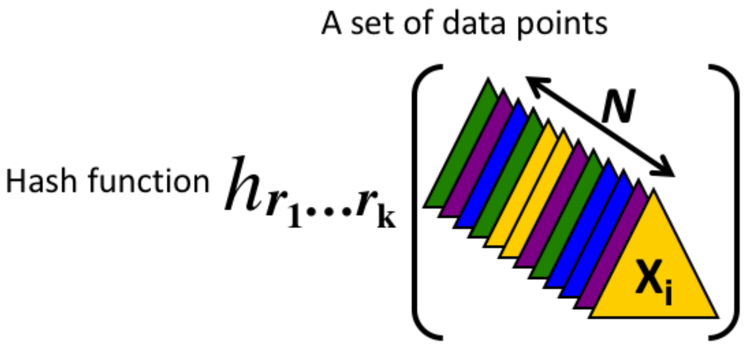


...	
100	a b d
101	
110	c
...	

...	
001	d
010	b
011	a c e
...	

Table 2

How to search from hash table?



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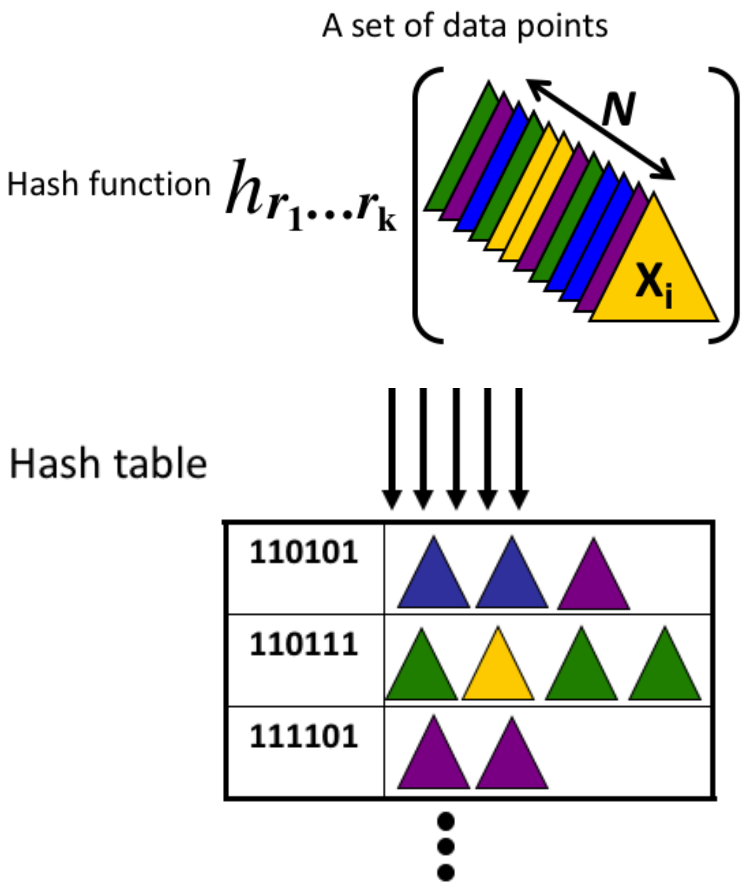

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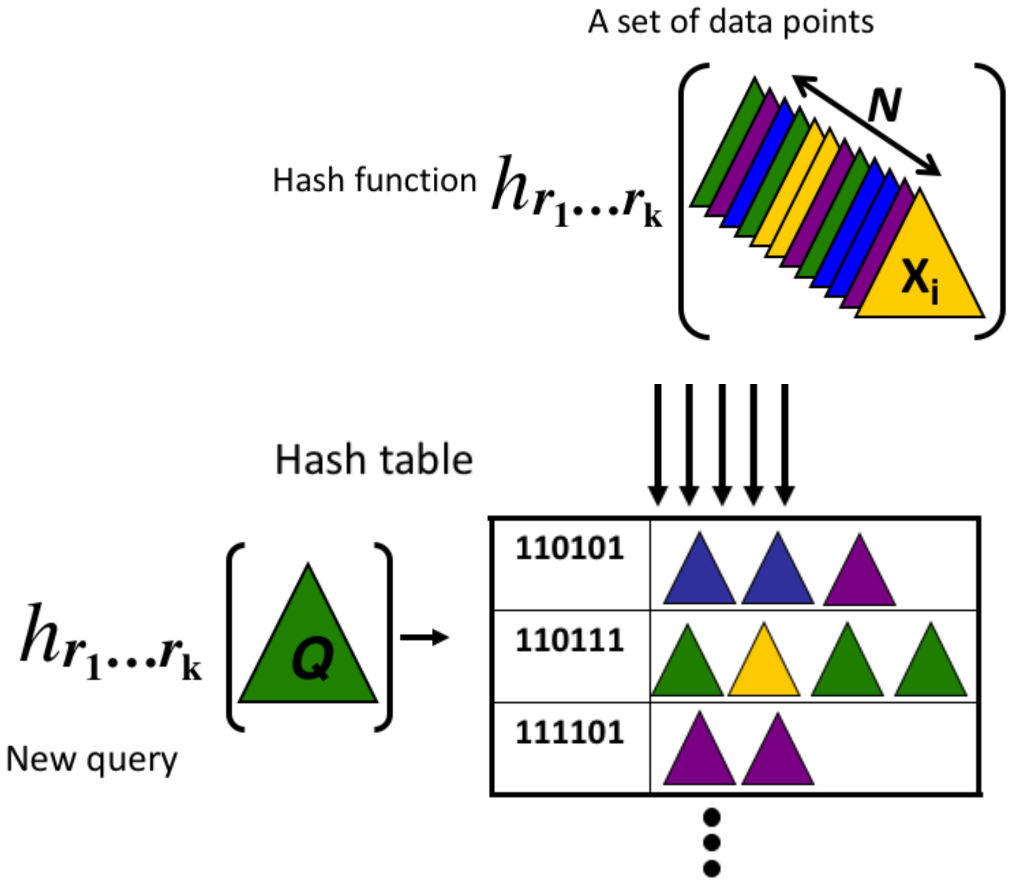

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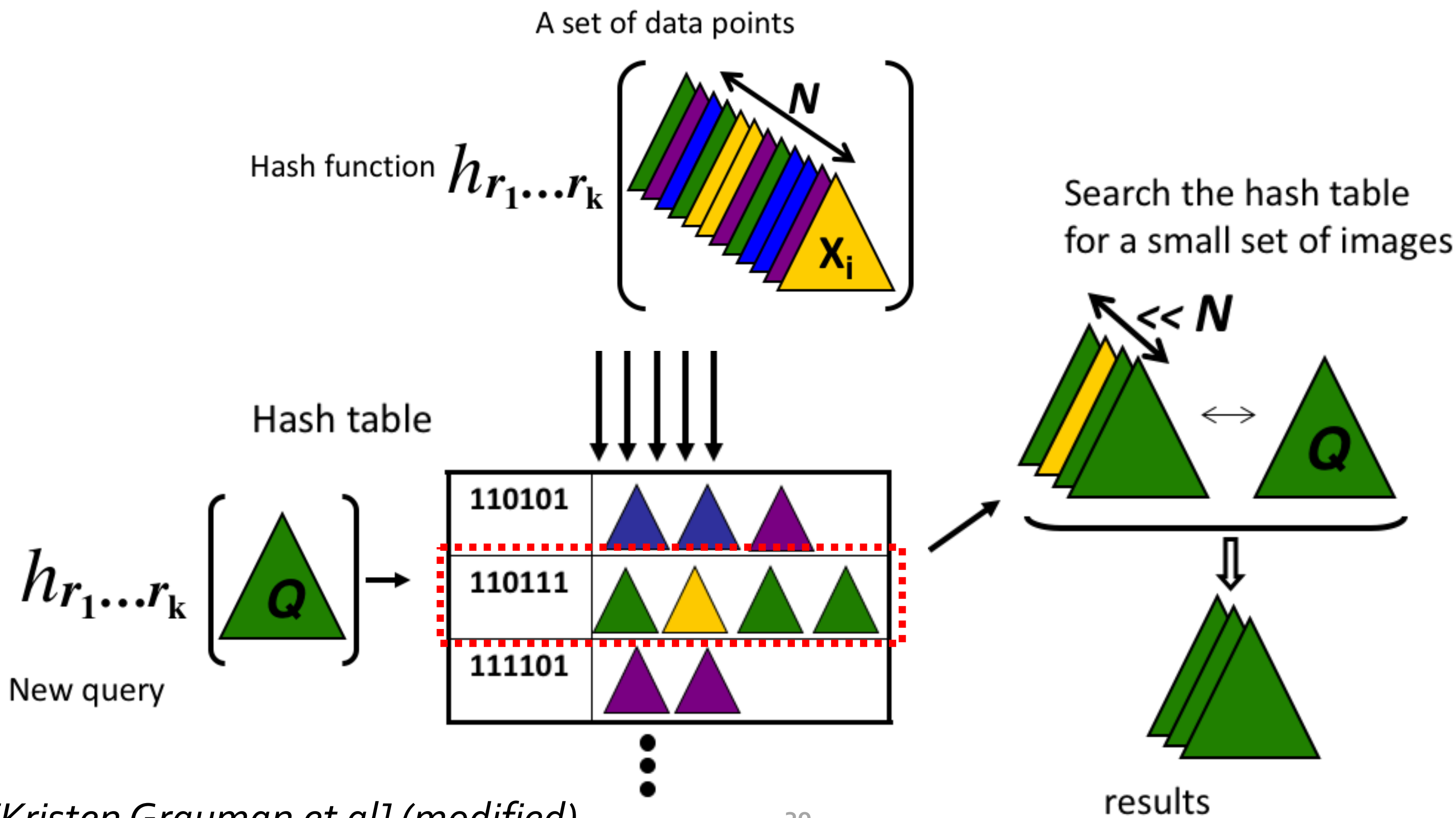
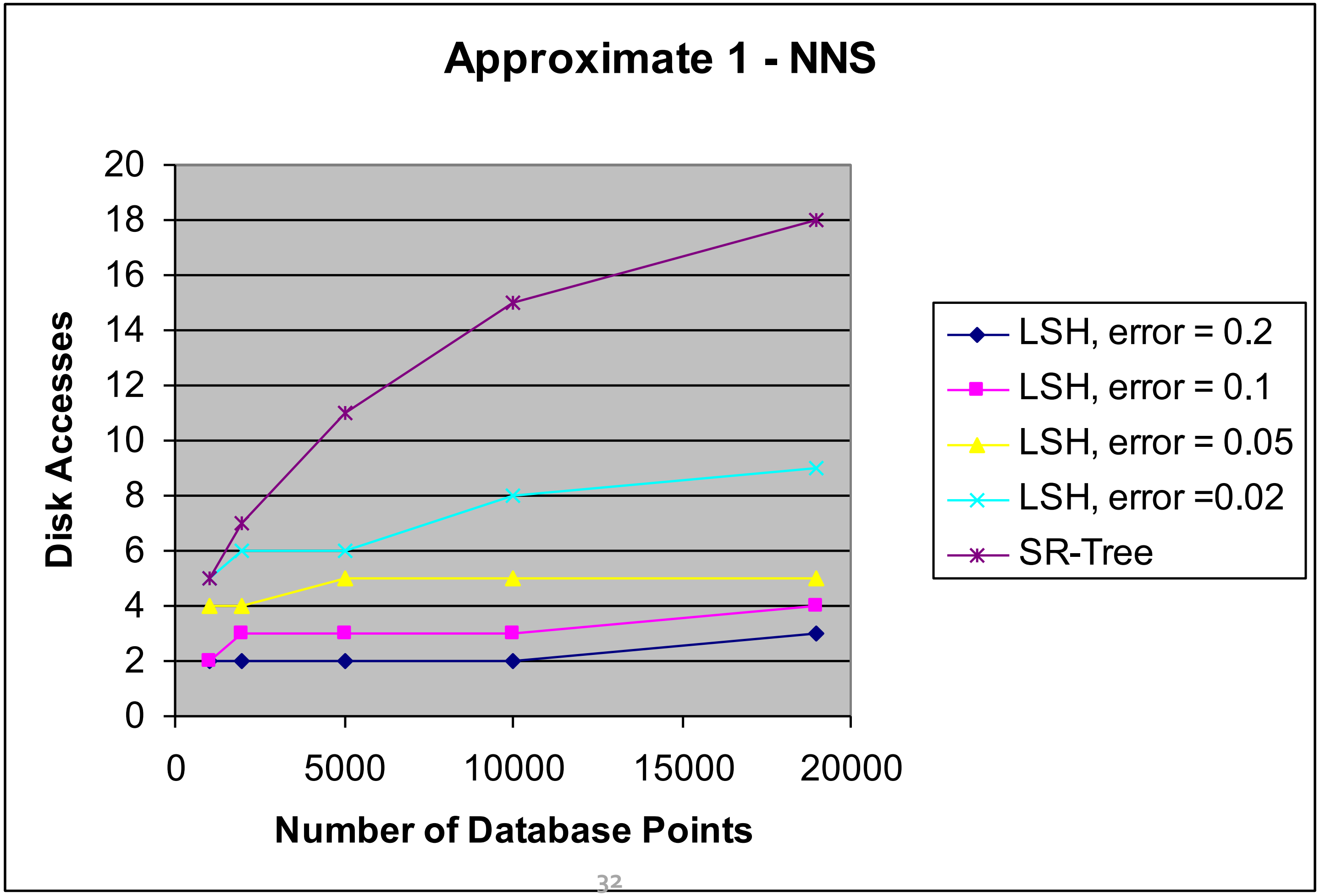


[Kristen Grauman et al] (modified)

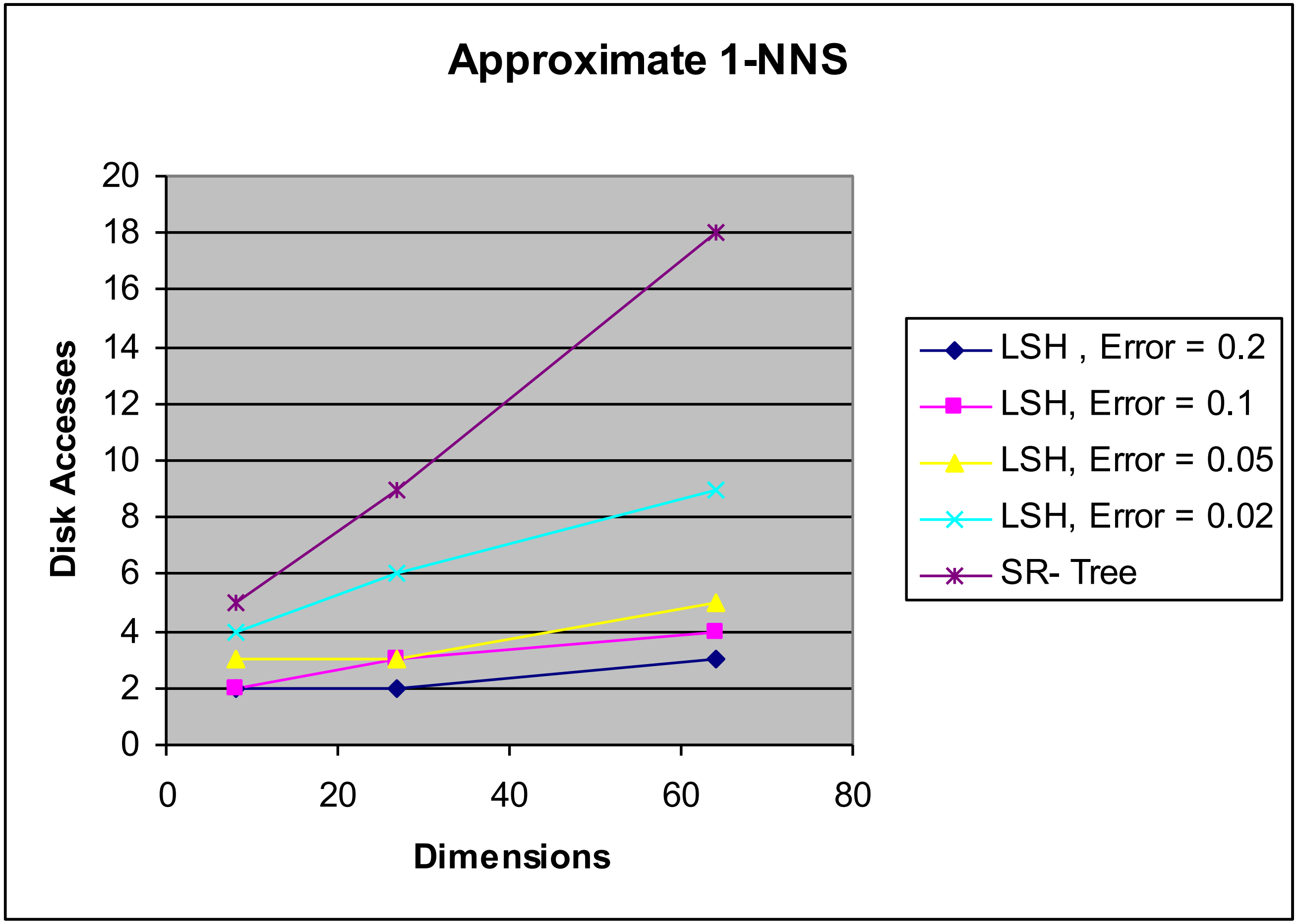
Performance

- Data Sets
  - Color images from COREL Draw library (20,000 points,dimensions up to 64)
  - Texture information of aerial photographs (270,000 points, dimensions 60)
- Evaluation
  - Speed, Miss Ratio, Error (%) for various data sizes, dimensions, and K values
  - Compare Performance with SR-Tree (Spatial Data Structure)

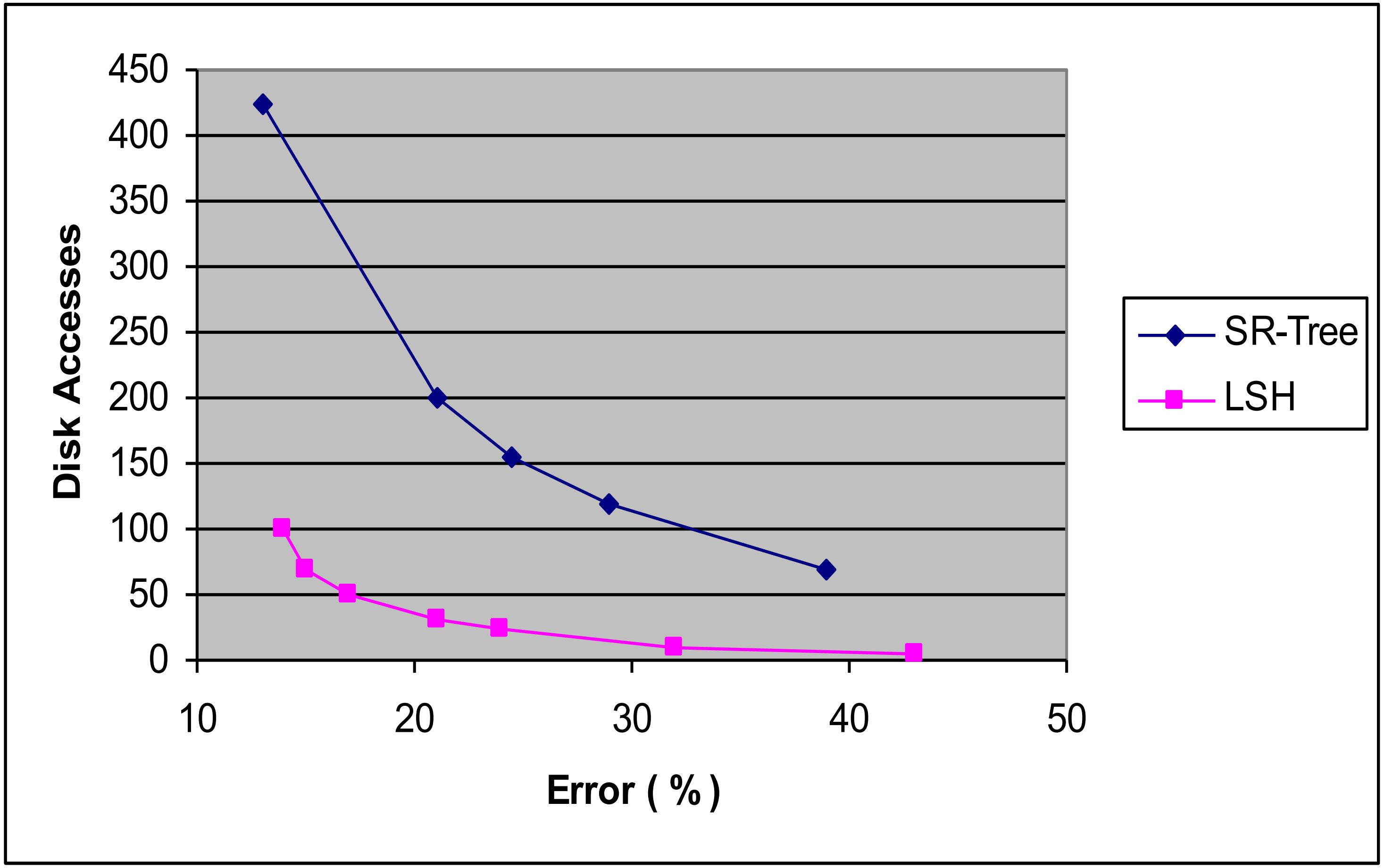
Speed vs. Data Size



Speed vs. Dimension



Speed vs. Error



Analysis

- LSH solves c-approximate NN with:
  - Number of hash functions: $L = \mathbf{n^{\rho}}$
  - $\boldsymbol{\rho} = \log(1/P_1)/\log(1/P_2)$
  - E.g., for the Hamming distance we have $\boldsymbol{\rho} = 1/c$
  - Constant success probability per query q
- Questions:
  - Can we extend this beyond Hamming distance?
  - Can we reduce the exponent $\boldsymbol{\rho}$?

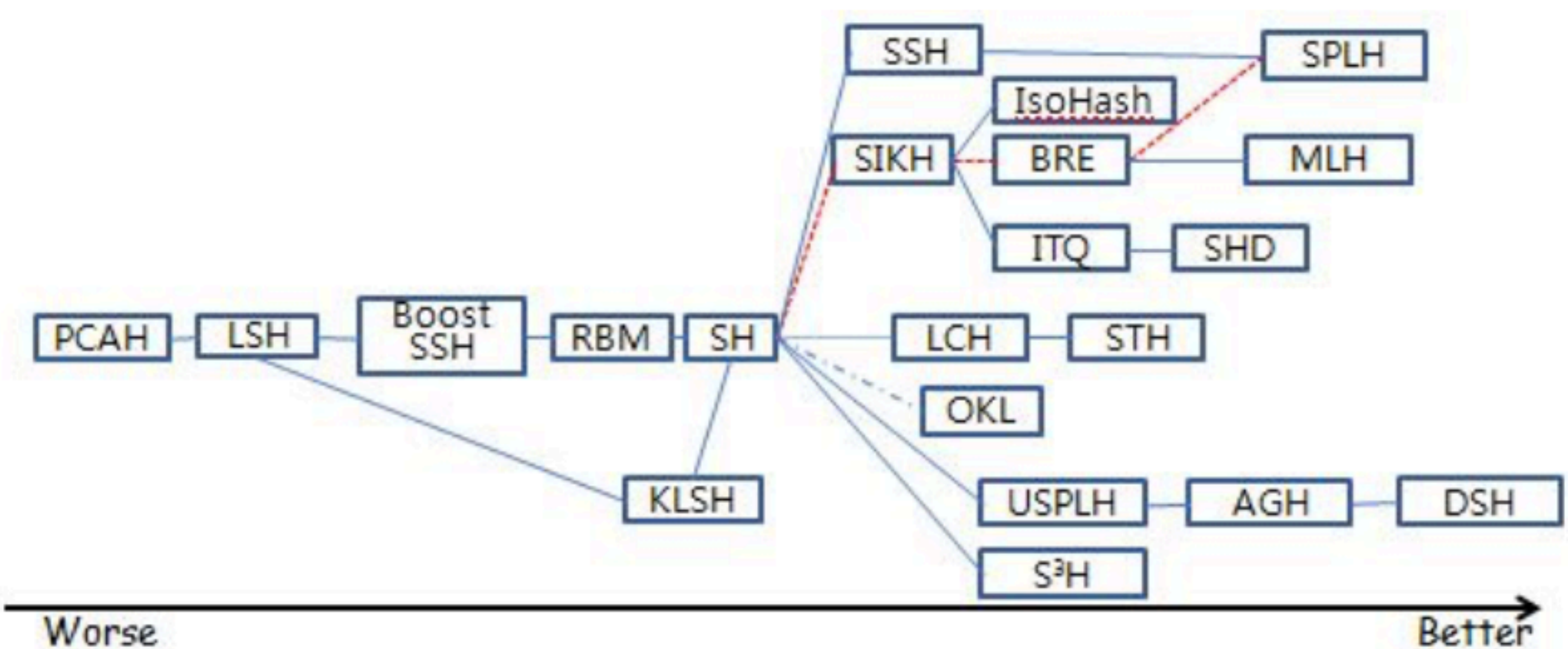
Analysis

Distance metric	$\rho = (\ln 1 / p_1) / (\ln 1 / p_2)$	c = 2	Reference
Euclidean (l_2)	$\leqq 1/c^2 + o(1)$	1/4	[Andoni, Indyk 2006]
	$\geqq 1/c^2 - o(1)$		[O'Donnell, Wu, Zhou 2011]
	$\dfrac{1}{2c^2 - 1} + o(1)$	1/7	[Andoni, R 2015]
Hamming (l_1)	$\leqq 1/c$	1/2	[Indyk, Motwani 1998]
	$\geqq 1/c - o(1)$		[O'Donnell, Wu, Zhou 2011]
	$\dfrac{1}{2c - 1} + o(1)$	1/3	[Andoni, R 2015]

Categorization of LSH approaches

- ✓ Number of bucket sets
- ✓ Shape of hash functions – (bit sampling, hyperplane, sine function, hypersphere)
- ✓ Data dependency
- ✓ Supervision
- ✓ Quantization - (single-bit quantization, multi-bits quantization)

Comparison [Lee, 2012]



Acknowledgments

PATREO
UFMG
30 ANOS FAPEMIG 1986 - 2016
CAPES
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Conselho Nacional de Desenvolvimento Científico e Tecnológico

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